ASYMPTOTIC SOLUTION OF WHIPPLE'S EQUATION FOR GAS LUBRICANT WITH HIGH COMPRESSIBILITY NUMBERS

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Gas-dynamic bearings that provide adequate carrying capacity without special injection of gas are widely employed in engineering. The gas flow through the working gap is created by grooving the part of the surface adjacent to one of the boundaries. The rest of the surface is smooth.

Assuming that the density of the grooves on the grooved part is large, the equation for the average pressure can be employed in the calculations [1]. This is a partial differential equation of the elliptic type, and a boundary-value problem of the first kind is formulated for it. The solution, however, has some properties that are characteristic for equations of the hyperbolic type, since it is obvious from physical considerations that the flow rate should largely be determined by the grooved part.

This paper is concerned with clarifying how the gas flow through the working gap is formed and how the parameters of the grooves affect the pressure and load distributions. An asymptotic expansion with respect to the compressibility parameter Λ ($\Lambda \to \infty$) is constructed. The gas flow rate in the limit $\Lambda \to \infty$ is determined completely by the parameters of the grooves at the input boundary and is of the order of Λ in the isothermal case; the pressure in the shaped part is of the order of unity. On the smooth part the pressure is of the order of $\Lambda^{1/2}$. The carrying capacity, determined from the asymptotic solution, is compared with the value determined by direct numerical solution of the starting problem for a spherical bearing.

1. We shall study the gas flow in a thin working gap in a sliding bearing (Fig. 1), whose top surface rotates around the symmetry axis of the bearing with an angular velocity ω. The stationary equation for the pressure in the thin gas layer (Reynolds equation) for a polytropic process has the form

$$\frac{\partial}{\partial x'^1}(q'^1\sqrt{g'}) + \frac{\partial}{\partial x'^2}(q'^2\sqrt{g'}) = 0,$$
$$q'^i = \frac{U_f'^i}{\sqrt{g'_{ii}}} h' p'^{1/\varkappa} - \frac{h'^3}{12\mu} p'^{1/\varkappa} \frac{\partial p'}{\partial x'^k} g'^{ik}, \quad i, k = 1, 2$$

(summation over k). Here $\varkappa \geqslant 1$ is the index of the polytrope; $U_f'^i$ are the physical components of the local rolling velocity vector (the half-sum of the velocities of the surfaces); h' is the film thickness; p' is the pressure; x'^1 and x'^2 are curvilinear coordinates, determining the position of a point on the surfaces separated by the working gap; g'^{ik} and g'_{ik} are the components of the metric tensor in curvilinear coordinates associated with one of the surfaces ($g' = g'_{11}g'_{22} - g'^2_{12}$); and, μ is the dynamic coefficient of viscosity. We introduce the characteristic scales p_a, h_0, L_0, and U_0 for the pressure, thickness, length, and velocity, respectively, and the scales L_1 and L_2 for the coordinates x'^1 and x'^2. We transform to dimensionless variables according to the formulas $x'^1 = x^1L_1$, $x'^2 = x^2L_2$, $p' = pp_a$, $h' = hh_0$, $U_f'^i = U_f^iU_0$, $ds' = dsL_0$ (ds' is a differential with the dimensions of length). In these variables the starting equation assumes the form

$$\frac{\partial}{\partial x^1}(q^1\sqrt{g}) + \frac{\partial}{\partial x^2}(q^2\sqrt{g}) = 0,$$
$$q^i = \Lambda\frac{U_f^i}{\sqrt{g_{ii}}} h p^{1/\varkappa} - h^3 p^{1/\varkappa}\frac{\partial p}{\partial x^k} g^{ik}, \tag{1.1}$$

where $g_{ik} = \frac{L_iL_k}{L_0^2} g'_{ik}$; $g^{ik} = \frac{L_0^2}{L_iL_k} g'^{ik}$; $g = g_{11}g_{22} - g_{12}^2$, and $\Lambda = 12\mu L_0U_0/(h_0^2p_a)$ is the compressibility number.

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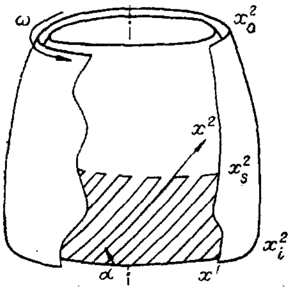

Fig. 1

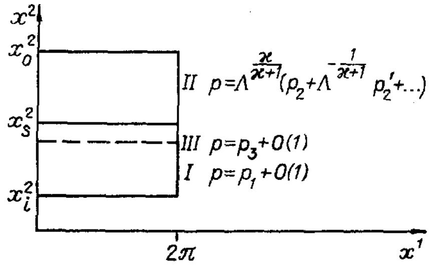

Fig. 2

To increase the pressure in the gap, part of one surface is grooved. The part of the surface occupied by the grooves is hatched in Fig. 1. We assume that the grooves are made on the inner (stationary) surface. In [1] Whipple's equation for the pressure averaged over the grooves was derived by an asymptotic method of two-scale expansions, analogous to the method of [2]:

$$\frac{\partial}{\partial x^1}(\langle q^1\rangle\sqrt{g})+\frac{\partial}{\partial x^2}(\langle q^2\rangle\sqrt{g})=0,$$

$$\langle q^i\rangle=\Lambda p^{1/\varkappa}A^i-p^{1/\varkappa}\frac{\partial p}{\partial x^k}G^{ik},\quad A^1=\frac{U_f^1}{\sqrt{g_{11}}}\frac{\langle h^{-2}\rangle}{\langle h^{-3}\rangle},\tag{1.2}$$

$$A^2=\frac{U_f^2}{\sqrt{g_{22}}}\langle h\rangle+\frac{U_f^1}{\sqrt{g_{11}}}\frac{g^{12}}{g^{11}}\left(\frac{\langle h^{-2}\rangle}{\langle h^{-3}\rangle}-\langle h\rangle\right),\quad G^{11}=\frac{g^{11}}{\langle h^{-3}\rangle},\ G^{12}=G^{21}=\frac{g^{12}}{\langle h^{-3}\rangle},$$

$$G^{22}=g^{22}\langle h^3\rangle-\frac{(g^{12})^2}{g^{11}}\left(\langle h^3\rangle-\frac{1}{\langle h^{-3}\rangle}\right).$$

Here $\langle y\rangle$ is the value of y averaged over the grooves (more accurately, over the fast coordinate, directed across the grooves). In the derivation of Eq. (1.2) the coordinate system was chosen so that the x^2 is oriented along the grooves.

For the sliding bearing shown in Fig. 1, $U_f^2 \equiv 0$, $U_f^1 > 0$, $g_{12} > 0$ (and therefore $g^{12} < 0$). The boundary conditions for (1.2) are: $p|_{x^2=x_i^2}=p^i(x^1)$, $p|_{x^2=x_0^2}=p^0(x^1)$, $p(0,x^2)=p(2\pi,x^2)$. On the boundary $x^2 = x_S^2$, separating the smooth and shaped regions, the pressure and the component of the flow normal to the boundary are required to be continuous; the last condition is equivalent to continuity of $\langle q^2\rangle$. The entire region occupied by the lubricant is determined by the inequalities $0\leqslant x^1\leqslant 2\pi$, $x_i^2\leqslant x^2\leqslant x_0^2$.

The solution of Eq. (1.1) in the limit $\Lambda\to\infty$ is presented in [1, 3]. We shall construct by the method of joined asymptotic expansions [4] the solution of the problem for Eq. (1.2) in the limit $\Lambda\to\infty$. We divide the region of the solution (Fig. 2) into three subregions. We assume that the pressure is of the order of unity in the regions I and III and of the order of $\Lambda^{\varkappa/(\varkappa+1)}$ in region II. The exponent $\varkappa/(\varkappa+1)$ is obtained from the condition that the orders of $\langle q^2\rangle$ be the same in regions II and III. We shall assume that the widths of the regions I and II are of the order of unity, while region III is a boundary layer and its width is asymptotically small.

Let us examine the region I. We substitute the expansion $p = p_1(x^1, x^2) + o(1)$ into Eq. (1.2) and, taking into account the fact that the size of this region is of the order of unity, we obtain in the leading order

$$\frac{\partial}{\partial x^1}(\sqrt{g}A^1p_1^{1/\varkappa})+\frac{\partial}{\partial x^2}(\sqrt{g}A^2p_1^{1/\varkappa})=0.\tag{1.3}$$

We introduce in the region III the interior variable $\eta=\Lambda(x^2-x_S^{\prime})$, where $x_S^{\prime}$ is the x^2 coordinate of a point in the region III. The equation for the pressure in this region is

$$\frac{\partial}{\partial\eta}\left[A^2(x^1,x_s^2)\,p_3^{1/\varkappa}-G^{22}(x^1,x_s^2)\,p_3^{1/\varkappa}\frac{\partial p_3}{\partial\eta}\right]=0.\tag{1.4}$$

Integrating it we find the solution in quadratures:

$$\int\limits_{p_3^0}^{p_3}\frac{dz}{1-z^{-1/\varkappa}f_3/A^2}=\frac{A^2(x^1,x_s^2)}{G^{22}(x^1,x_s^2)}\eta_\bullet\tag{1.5}$$

where f_3 is the constant of integration of Eq. (1.4) with respect to η, and $p_3^0 = p_3|_{\eta=0}$. In the limit $\eta \to -\infty$ the solution (1.5) must approach, by virtue of the joining, the limiting value $p_1(x^1, x_S^2)$. One can see from (1.5) that this is possible only if $f_3 = A^2(x^1, x_s^2)\, p_1^{1/\varkappa}(x^1, x_s^2)$; in this case the integral on the left side converges for $p_3 = p_1(x^1, x_S^2)$. Since $p_1 > 0$ and $A^2 > 0$ (this inequality will be proven below), $f_3 > 0$. For $\eta = \eta_S = \Lambda(x_S^2 - x_S^1)$ the condition of continuity of the pressure is satisfied, i.e.,

$$p_3(x^1, \eta_s) = \Lambda^{\varkappa/(\varkappa+1)} p_2(x^1, x_s^2). \tag{1.6}$$

The leading part of the integral (1.5) for large values of p_3 equals p_3. For this reason it follows from (1.5) and (1.6) that $\eta_s = \Lambda^{\varkappa/(\varkappa+1)} \dfrac{G^{22}(x^1, x_s^2)}{A^2(x^1, x_s^2)}\, p_2(x^1, x_s^2) + o(\Lambda^{\varkappa/(\varkappa+1)})$ and in the variables of the boundary layer the start of the region II shifts to $+\infty$. It follows from (1.4) that the component of the flow rate vector $\langle q^2 \rangle$ is conserved in the boundary layer, and from (1.2) and (1.4) it follows that $\langle q^2 \rangle = \Lambda A^2 p_1^{1/\varkappa}(x^1, x_s^2) + o(\Lambda)$.

In the region II, where there are no grooves, the problem is formulated as follows. We are required to solve Eq. (1.1) [which can be formally derived from (1.2) with the substitution $\langle y \rangle = y$] with the above-indicated boundary conditions at $x^2 = x_0^2$, $x^1 = 0$, $x^1 = 2\pi$ and with the component of the flow vector $\langle q^2 \rangle$ fixed on the boundary $x^2 = x_S^2$:

$$-\Lambda h^3 p_2^{1/\varkappa}\left(g^{21}\frac{\partial p_2}{\partial x^1} + g^{22}\frac{\partial p_2}{\partial x^2}\right)\Big|_{x^2=x_s^2} = \Lambda A^2 p_1^{1/\varkappa}\big|_{x^2=x_s^2} + o(\Lambda).$$

Substituting the expansion of the pressure in the region II into (1.1) and applying the boundary conditions, we obtain the problem for p_2:

$$\begin{gathered}\frac{\partial}{\partial x^1}\left(\frac{\sqrt{g}\,U_f^1}{\sqrt{g_{11}}} h p_2^{1/\varkappa}\right) = 0,\\ -h^3 p_2^{1/\varkappa}\left(g^{21}\frac{\partial p_2}{\partial x^1} + g^{22}\frac{\partial p_2}{\partial x^2}\right)\Big|_{x^2=x_s^2} = A^2(x^1, x_s^2)\, p_1^{1/\varkappa}(x^1, x_s^2),\\ p_2(0, x^2) = p_2(2\pi, x^2), \quad p_2 = (x^1, x_0^2) = 0.\end{gathered} \tag{1.7}$$

From the first equation we have

$$p_2 = \left(\frac{f_0(x^2)\sqrt{g_{11}}}{U_f^1 h\sqrt{g}}\right)^{\varkappa}, \tag{1.8}$$

where $f_0(x^2)$ is an unknown function. To determine it we shall study the equation for p_2' obtained from (1.1):

$$\frac{\partial}{\partial x^1}\left[\sqrt{g}\left(\frac{U_f^1}{\varkappa\sqrt{g_{11}}} h p_2^{(1-\varkappa)/\varkappa} p_2' - h^3 p_2^{1/\varkappa}\left(g^{11}\frac{\partial p_2}{\partial x^1} + g^{12}\frac{\partial p_2}{\partial x^2}\right)\right)\right] - \frac{\partial}{\partial x^2}\left[\sqrt{g}\, h^3 p_2^{1/\varkappa}\left(g^{21}\frac{\partial p_2}{\partial x^1} + g^{22}\frac{\partial p_2}{\partial x^2}\right)\right] = 0. \tag{1.9}$$

Integrating it over x^1 and using the condition $p_2'(0, x^2) = p_2'(2\pi, x^2)$ we find

$$-\int_0^{2\pi}\sqrt{g}\, h^3 p_2^{1/\varkappa}\left(g^{21}\frac{\partial p_2}{\partial x^1} + g^{22}\frac{\partial p_2}{\partial x^2}\right) dx^1 = C. \tag{1.10}$$

It follows from the second relation in (1.7) that

$$C = \int_0^{2\pi}\sqrt{g(x^1, x_s^2)}\, A^2(x^1, x_s^2)\, p_1^{1/\varkappa}(x^1, x_s^2)\, dx^1. \tag{1.11}$$

Using the formula (1.8) and (1.10), we obtain an ordinary differential equation for the unknown function f_0:

$$A\frac{df_0^{\varkappa+1}}{dx^2} + Bf_0^{\varkappa+1} = -C,$$

$$A(x^2) = \frac{\varkappa}{\varkappa+1}\int_0^{2\pi} h^3\sqrt{g}\, g^{22}\left(\frac{\sqrt{g_{11}}}{\sqrt{g}\,U_f^1 h}\right)^{\varkappa+1} dx^1,$$

$$B(x^2)=\varkappa\int_0^{2\pi}\frac{h^2\sqrt{g_{11}}}{U_j^1}\left(\frac{\sqrt{g_{11}}}{\sqrt{g}\,U_j^1h}\right)^{\varkappa-1}\left[g^{21}\frac{\partial}{\partial x^1}\left(\frac{\sqrt{g_{11}}}{\sqrt{g}\,U_j^1h}\right)+g^{22}\frac{\partial}{\partial x^2}\left(\frac{\sqrt{g_{11}}}{\sqrt{g}\,U_j^1h}\right)\right]dx^1. \tag{1.12}$$

The solution of (1.12) satisfying the fourth condition in (1.7) is

$$f_0^{\varkappa+1}=-C\int_{x_0^2}^{x^2}\frac{1}{A}\exp\left(\int_{x_0^2}^{x^{2\prime}}\frac{B}{A}\,dx^{2''}\right)dx^{2'}\exp\left(-\int_{x_0^2}^{x^2}\frac{B}{A}\,dx^{2'}\right).$$

Thus the pressure in the region II in leading order is determined by the formula

$$p=\Lambda^{\varkappa/(\varkappa+1)}\left(\frac{\sqrt{g_{11}}}{\sqrt{g}\,U_j^1h}\right)^{\varkappa}\left[\int_0^{2\pi}\sqrt{g(x^1,x_s^2)}\,A^2(x^1,x_s^2)\times\right.$$

$$\left.\times\, p_1^{1/\varkappa}(x^1,x_s^2)\,dx^1\right]^{\varkappa/(\varkappa+1)}\left(\int_{x_0^2}^{x^2}\frac{1}{A}\exp\left(\int_{x_0^2}^{x^{2\prime}}\frac{B}{A}\,dx^{2''}\right)dx^{2'}\right)^{\varkappa/(\varkappa+1)}\left[\int_{x_0^2}^{x^2}\frac{1}{A}\exp\left(\int_{x_0^2}^{x^{2\prime}}\frac{B}{A}\,dx^{2''}\right)dx^{2'}\right]^{\varkappa/(\varkappa+1)}. \tag{1.13}$$

We note that in the leading order approximation there was no need to study the boundary layer at the boundary $x^2 = x_0^2$, since the exterior solution satisfied the boundary condition.

The expression (1.13) contains the quantity $p_1(x^1, x_s^2)$, which is determined from the solution of Eq. (1.3) with the boundary condition on the pressure imposed on the boundary $x^2 = x_i^2$. The fact that the boundary layer in the grooved region does not arise at the boundary $x^2 = x_i^2$ is a direct consequence of the inequality $A^2 > 0$. Indeed, let us assume that the boundary layer is formed on the boundary $x^2 = x_i^2$. Then the relation (1.5) will be satisfied, the only difference being that joining occurs in the limit $\eta \to +\infty$. Substituting the value of f_3 and differentiating (1.5), we obtain $\frac{dp_3}{d\eta}=\frac{A^2}{G^{22}}\times\frac{p_3^{1/\varkappa}-p_1^{1/\varkappa}}{p_3^{1/\varkappa}}$. p_3 can be joined with p_1 in the limit $\eta \to +\infty$ only if $p_3 \equiv p_1$, since the fact that p_3 differs from p_1 for some η will cause it to grow without bound at infinity. Thus joining is possible only in the limit $\eta \to -\infty$, and this is what caused the boundary layer to form at the boundary $x^2 = x_s^2$.

2. We shall now prove the inequality $A^2 > 0$. The average value of y is found by averaging $\left(\langle y\rangle=\lim_{Y\to\infty}\int_0^Y\frac{1}{Y}\,y\,d\xi\right)$ over the fast variable ξ. We shall prove that for a strictly positive function $y(\xi)$, where $\xi \in [0, 1]$, the inequality

$$Q=\gamma\beta\left(\int_0^1 y^{\gamma+\beta}d\xi-\int_0^1 y^{\gamma}d\xi\int_0^1 y^{\beta}d\xi\right)\geqslant 0 \tag{2.1}$$

holds. We transform the left side into the form $Q=\gamma\beta\int_0^1\int_0^1(y^{\gamma+\beta}(\xi)-y^{\gamma}(\xi)\,y^{\beta}(\zeta))d\xi\,d\zeta$. In the expression obtained we exchange the variables of integration and add the two expressions:

$$Q=\frac{\gamma\beta}{2}\int_0^1\int_0^1[-y^{\gamma}(\xi)\,y^{\beta}(\zeta)-y^{\gamma}(\zeta)\,y^{\beta}(\xi)+y^{\gamma+\beta}(\xi)+y^{\gamma+\beta}(\zeta)]\,d\xi\,d\zeta.$$

We note that the integrand equals $[y^{\gamma}(\xi)-y^{\gamma}(\zeta)][y^{\beta}(\xi)-y^{\beta}(\zeta)]$. Its sign is the same as that of the product $\gamma\beta$. The inequality (2.1) is thereby proved. Performing some simple transformations, we obtain

$$(\langle y^{\gamma+\beta}\rangle-\langle y^{\gamma}\rangle\langle y^{\beta}\rangle)\gamma\beta\geqslant 0. \tag{2.2}$$

If it is assumed in addition that y is a periodic, piecewise-continuous function with nonzero support, it can be shown that for $\gamma\beta \neq 0$ in (2.2) the strict inequality holds. This assumption holds for profiles of the film thickness $h(\xi)$.

Thus setting $\gamma = 1$ and $\beta = -3$ we obtain $\langle h^{-2}\rangle/\langle h^{-3}\rangle - \langle h\rangle < 0$, and therefore $A^2 > 0$.

3. We shall construct a solution for a spherical bearing with spiral grooves with purely axial displacement, i.e., in the case $\partial/\partial x^1 \equiv 0$. We denote the radius of the bearings by R,

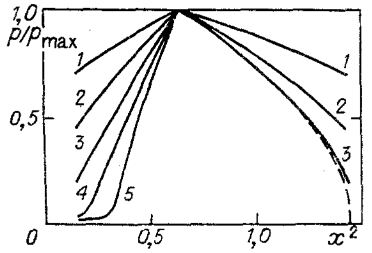

Fig. 3

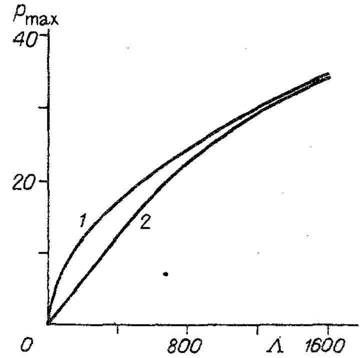

Fig. 4

and its angular rotational velocity by ω. Let θ be the angle measured from the positive direction on the axis of rotation, and φ the longitude, measured in the direction of rotation. The groove makes an angle $\alpha > 0$ with the latitude.

We introduce $\lambda = \ln|\tan(\theta/2)|$; then the equation of the line oriented along the groove is $\varphi + \lambda \cot\alpha = \text{const}$.

We introduce $x^2 = -\lambda$, $x^1 = \varphi + \lambda \cot\alpha$. We set $L_i = 1$, $i = 1, 2$, $L_0 = R$, $U_0 = R\omega/2$; then $U_j^1 = 1/\cosh x^2$. Let the pressure at the boundaries $x^2 = x_i^2$, $x^2 = x_0^2$ equal atmospheric pressure p_a; then $p^i(x^1) = p^0(x^1) = 1$.

The following relations hold:

$$g_{ik} = \frac{1}{\mathrm{ch}^2 x^2}\begin{pmatrix} 1 & \mathrm{ctg}\,\alpha \\ \mathrm{ctg}\,\alpha & \dfrac{1}{\sin^2\alpha}\end{pmatrix}, \quad g^{ik} = \mathrm{ch}^2 x^2 \begin{pmatrix} \dfrac{1}{\sin^2\alpha} & -\mathrm{ctg}\,\alpha \\ -\mathrm{ctg}\,\alpha & 1\end{pmatrix}, \tag{3.1}$$
$$g = 1/(\mathrm{ch}^4 x^2), \quad \mathrm{ch}^{-1}\lambda = \sin\theta.$$

It follows from (1.3) that $\sqrt{g}\,A^2 p_1^{1/\varkappa} = \sqrt{g(x_i^2)}\,A^2(x_i^2)$. The variable x^1 here and below is omitted.

The quantity on the right side of the first relation in (1.7) equals $A^2(x_s^2)\,p_1^{1/\varkappa}(x_s^2) = A^2(x_i^2)\sqrt{\dfrac{g(x_i^2)}{g(x_s^2)}}$. The following expressions are obtained for $A(x^2)$ and $B(x^2)$ in (1.12):

$$A(x^2) = \frac{2\pi\varkappa}{\varkappa+1}(\mathrm{ch}\,x^2)^{2\varkappa+2}(h(x^2))^{2-\varkappa},$$
$$B(x^2) = -2\pi\varkappa h^2\,\mathrm{ch}^2 x^2\left(\frac{\mathrm{ch}^2 x^2}{h}\right)^{\varkappa+1}\frac{d}{dx^2}(h\,\mathrm{ch}^{-2}x^2),$$
$$\frac{B}{A}(x^2) = -(\varkappa+1)\frac{d}{dx^2}\left[\ln\left(\frac{h}{\mathrm{ch}^2 x^2}\right)\right],$$
$$\int_{x_0^2}^{x^2}\frac{1}{A}\exp\left(\int_{x_0^2}^{x^{2\prime}}\frac{B}{A}\,dx^{2\prime\prime}\right)dx^{2\prime} = \frac{\varkappa+1}{2\pi\varkappa}\left(\frac{h(x_0^2)}{\mathrm{ch}^2 x_0^2}\right)^{\varkappa+1}\int_{x_0^2}^{x^2}\frac{dx^2}{h^3(x^2)}.$$

The pressure in the smooth part is determined by the formula

$$p = \Lambda^{\varkappa/(\varkappa+1)}\left[\frac{\varkappa+1}{\varkappa}\sqrt{g(x_i^2)}\,A^2(x_i^2)\int_{x_0^2}^{x^2}\frac{dx^2}{h^3(x^2)}\right]^{\varkappa/(\varkappa+1)}, \tag{3.2}$$
$$A^2 = \sin\alpha\cos\alpha(\langle h^{-2}\rangle/\langle h^{-3}\rangle - \langle h\rangle).$$

Thus in the limit $\Lambda \to \infty$ the carrying capacity is created primarily by the smooth part. The grooves cause the grooved part to drag out the flow by an amount of the order of Λ. The smooth part, however, because of the fact that there is no convective term in the expression q_2 on it, is a kind of stop. This is what is responsible for the high pressures of the order of $\Lambda^{\varkappa/(\varkappa+1)}$.

Figure 3 shows the computer calculations of the pressure distribution in the spherical support with a constant nominal gap $h = 2\cdot10^{-6}$ m, a radius of the spheres $R = 9\cdot10^{-3}$ m, $\varkappa = 1$ (isothermal flow), $\alpha = 30°41'$, groove depth in the grooved part $\Delta h = 4\cdot10^{-6}$ m, the relative groove widths 0.603, $x_i^2 = 0.1455$, $x_S^2 = 0.6344$, $x_0^2 = 1.44$. The curves 1-5 correspond to Λ = 13.7, 41, 136, 957, and 1777; the broken lines correspond to the limiting solution on the smooth part. The curves 4 and 5 on this section are virtually identical with the limiting curve and are not shown in the figure.

As follows from (3.2), with h = const on the smooth part $p = \Lambda^{1/2}\left[\frac{\sin 2\alpha}{\mathrm{ch}^2 x_i^2}\left(\langle h\rangle - \frac{\langle h^{-2}\rangle}{\langle h^{-3}\rangle}\right)\Big|_{x^2=x_i^2}\right]^{1/2}\frac{\sqrt{x_0^2-x^2}}{h^{3/2}}$. One can see that the pressure and therefore also the carrying capacity are maximum for $\alpha = 45°$. It also follows from the formula that the pressure p reaches the maximum value p_{max} at $x^2 = x_S^2$.

Figure 4 shows the two dependences $p_{max}(\Lambda)$ (1: the reduced asymptotic solution; 2: the result of the computer solution of the complete problem).

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STRUCTURE OF SHOCK WAVES IN POROUS IRON AT LOW PRESSURES

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The interest shown in the study of the behavior of porous materials under shock loading is due to their practical application in the explosive compaction of parts [1], their use in various types of shock-wave dampers [2], and the possibility such investigation offers for realizing a broad range of thermodynamic states in substances [3, 4].

The high-pressure region of shock compression, above 10 GPa, has traditionally been studied more intensively. This is due to the rapid strides made in shock-wave physics in recent years. In the low-pressure region – where the most important mechanical effects are realized in terms of the strength and plastic flow of a material in pores – relatively little information has been collected. The data that is available is restricted to isolated materials and porosities, and the results are often contradictory [2].

The well-known models of the mechanical behavior of porous materials fall into two groups: equilibrium models with an explicit $p \sim \rho$ relation [5, 6], and nonequilibrium models reflecting the kinetics of pore collapse [7-10].

Here, on the basis of the thermomechanical principles of a continuum with internal state parameters, we propose a model of the behavior of porous solids under shock loading. The results of mathematical modeling are compared with experimental measurements we made of the compression-wave profile in porous iron at different initial porosities (10-40%). The profiles were obtained by means of pressure gauges.

1. Description of the Model. The mechanics of deformable porous solids are based on several hypotheses, the most important of which are the hypothesis of continuity and the postulate of macroscopic definability [11].

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